

Stochastic Resonance in neural network, noise color effects.

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Some systems have phenomena that cannot be understood and predict by classical theory, that is why an explanation is found by a mixture of deterministic and stochastic theories. In order to achieve that, it is used noisy signals. Noise is not an interference signal that perturb the system, to the contrary, noise can help in the enhancement and understanding of the system functioning if the phenomena of nonlinear mechanics called Stochastic Resonance (SR) is study in detail. To detect this phenomena it is necessary that the system have a bistable potential barrier that creates a threshold, the input of the system should be a weak periodic signal which amplitude is below threshold together with an stochastic signal. In this way, the SR is detected when there are weak periodic signals that are added to different noise colors in order to be amplified and optimised. The interactions between the two signals transform the potential of the system precisely at the frequency of the weak periodic signal that is added to the system. The behaviour of the SR is detected in a neural network and it is study under noise color variations where Pink noise amplified the signal many orders of magnitude more than white noise.

I. INTRODUCTION

It is clear that a big part of physics, theoretical and experimental, has tried to do linear approximations to solve problems, without doubt, this approximations have given satisfactory results because of their coincidence with experimental findings to a good error margin, however, the measurements are subject to the same linear approximations and most of the time, data considered as noise is not considered, not because of its importance but because it is hard to analyse given our limited theoretical tools.

Real physical phenomena is intrinsically non lineal and its dynamics is modulated by a noisy environment. The clearest example is the nervous system which is embedded under all kinds of interactions with the external world and which shape its perception.¹.

It is essential to study the non lineal mechanics phenomena called Stochastic Resonance, this is completely different to the resonance given in lineal systems, the biggest difference is the frequency value that sets the resonance conditions, for the SR case, the resonance frequency is given by the weak periodic input signal, while for the traditional lineal resonance, the resonance frequency depends on the structural properties of the system.

One of the biggest applications to the study of the SR is that it is found in sensory systems and it is possible to understand how a stochastic process given by noise when added to a small perturbation stimulus in the subthreshold regime of the system, can amplify and enhance the information sensory process and perception.².

Something fundamental and the main characteristic of the non lineal systems is the requirement of three

components: a threshold³, the stimulus of the system with a weak signal that is in the subthreshold regime, and the addition of a noisy signal. These are the basic components that are always found in nature which make it possible its interaction and the facilitation for SR.

There is also evidence of the rol of SR in the functioning of the brain for the detection of weak signals, synchronisation and coherence in neural connections, synapses and behaviour in general⁴.

It is possible to understand the dependence of the stochastic resonance on the parameters of periodic perturbation, the noise and the system through the theoretical explanations that when it come to stochastic differential equations and non lineal equations, there is no analytical solution but could be appreciated with numerical simulations.

On another hand, the SR phenomena is also found in different artificial tools that have an intrinsic error threshold³ that can have serious complications during measurements, this makes it necessary that this error level could be reduced to the maximum to obtain an optimum function. It is possible to reduce this error level by the use of stochastic resonance³.

The following methods are used in order to analyse the SR phenomena:

- The Signal to Noise Ratio (SNR)
- The Inter-Spike Interval Histogram (ISIH)

These methods indicate the point of maximum SR to a given noise value and it is reflected the fact that the information of the input periodic signal its preserved and amplified.

Finally, given that noise is classified in colors, it is possible to generate comparisons of the presence of the

phenomena to different noise colors and in this way find the value and noise color for which the amplification is maxima.

The main motivation of this research is to understand how neurons make use of the SR phenomena and how is it possible to manipulate and change the presence of SR to conclude which are the consequences for signal transmission and perception⁵. In despite of the fact that measurements and data analysis have been done with an electronic device of an artificial neuron, the long term objective is to analyse the phenomena in biological neurons.

It is possible to see the phenomena in neurons given that its function in the brain is the integration and information processing of electrochemical signals, this constant activity at the interior of the brain creates a background activity given the neuronal pulses and the signal transmission that causes fluctuations in the membrane potential creating a real source of noise⁶. This noise is the responsible for signal amplification by Stochastic Resonance.

II. THEORETICAL BACKGROUND

SR can be understood in a simple way through the use of a model based in a brownian particle, the only additional component is the weak periodic force and the bistable potential, in this case the equation of motion is⁷:

$$m \frac{d^2x}{dt^2} = F(t) - b \frac{dx}{dt} - \frac{dV}{dx} + \xi(t) \quad (1)$$

Where $F(t)$ is the periodic force, $\xi(t)$ is the noisy signal and $V(x)$ is the bistable potential that is written in the following way³:

$$V(x) = -a \frac{x^2}{2} + b \frac{x^4}{2} \quad (2)$$

Where a and b are constants that can be manipulated to shape the potential as required. The potential can be seen in fig.1.

The constants a and b are related with the potential dimensions in the following way:

$$x_m = \sqrt{\frac{a}{b}} \quad \Delta V = \frac{a^2}{4b} \quad (3)$$

First of all the weak periodic signal is introduced to the system, this signal is not strong enough to move the particle from one side of the potential to the other, this case can be observed in fig.2.

After the addition of the weak periodic signal into the system, the noisy signal is added, and this signal allows that the particle could cross from one side of the potential to the other, amplifying in some way the particle movement when it was only stimulated by the

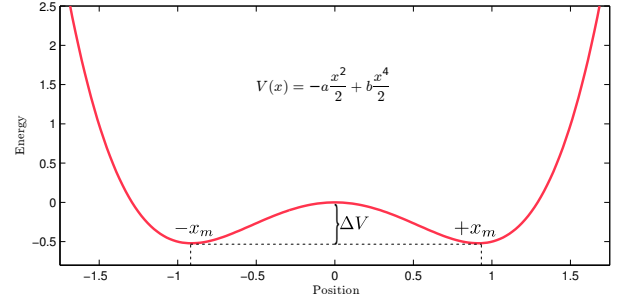


Figure 1. Bistable potential with which it is possible to model SR if a brownian particle is interacting with it.

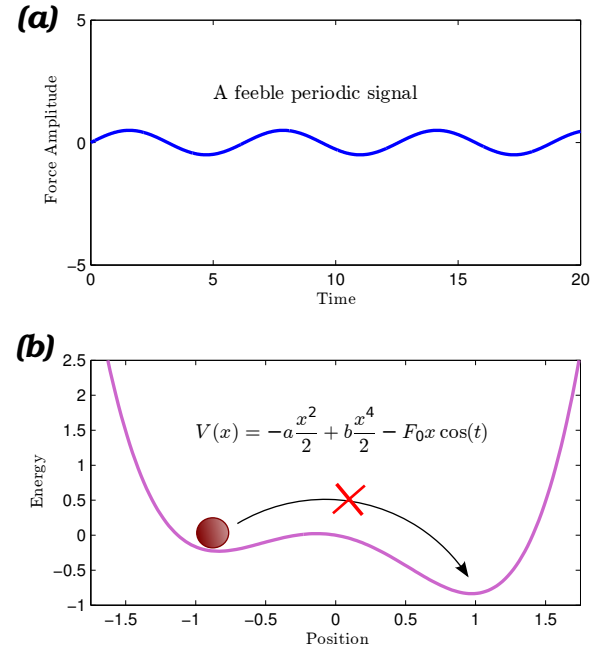


Figure 2. A weak periodic signal force the particle, its amplitude is not enough to make the particle move from one side of the bistable potential to the other.

weak signal. This situation can be seen in fig.3. The answer of the system depends on the noise intensity, because it is hypothesised that all amplitudes are convenient to the system. If the noise signal is small in amplitude fig. 4 (a) it is expected that not always achieves the crossing fig. 4(b) if the noise amplitude is too big (and it should be enormous as we will see in results) see fig. 4 (e) the signal finally will loose given that the particle will move from one side to the other of the potential, fig. 4 (f) which will not make possible to distinguish any coherent frequency. However, noise amplitude is convenient fig. 4 (c) and it will happen that crossing of the particle from one side of the potential to the next will be couple with the coherent signal fig.4(d) and it is here where Stochastic Resonance occurs.

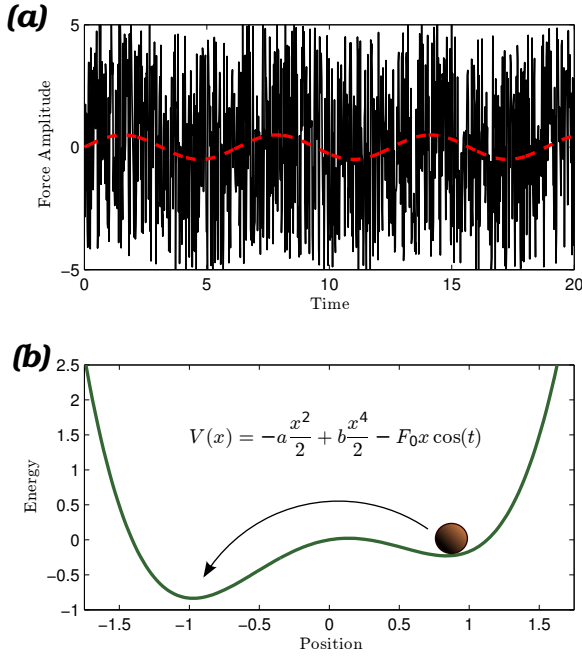


Figure 3. A noisy signal is added to the weak signal, with this additional signal the particle is able to cross from one side of the potential to the next.

The stochastic resonance that we analysed before happens in many systems perhaps the more relevant where this happens is in neurons. Neurons are excitable systems, this means that they have a threshold to emit a signal and the answer signal is represented by a peak similar to a δ , the time signal that a neuron emits is pretty similar to what is observed in fig.5.

A way to quantify SR in a neuron is by measuring the inactivity time as can be appreciated in fig. 5, then a lot neuron's response data is collected over time and the inactivity intervals are saved for future processing⁶.

Once the inactivity time have been collected, it is useful to organised them in a histogram, see fig. 6, in the histogram the time interval are organised in such a way that the maximums are at the times that correspond to the multiples of the input signal period. With the variation of the noise there is a change in the histogram. For small values of noise's amplitude, the times at which the phenomena occurs are pretty long and the majority of occurrences would happen for times bigger than the signal's period, to the contrary, if the noisy signal is too high, the inactivity times would be too short and the occurrences in the histogram would be accumulated before the time corresponding to the signal's period. Finally, if the noise's amplitude is adequate, occurrences would be accumulated over the interval that corresponds to the period of the signal, see fig. 6. SR is then quantified for the maximum which happens at the value

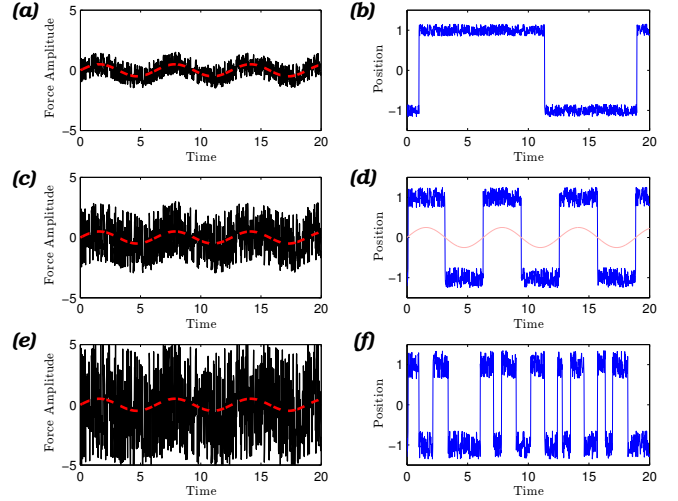


Figure 4. Response of the system, (a) Noise amplitude is weak, (b) then the crosses from one side of the potential to the other are not often, (c) the noise amplitude is optimum (d) and the cross from one side of the potential to the other is synchronised with the weak signal. (e) The noise amplitude is too high, (f) then the jumps are so frequent that all kind of information of the input signal is lost.

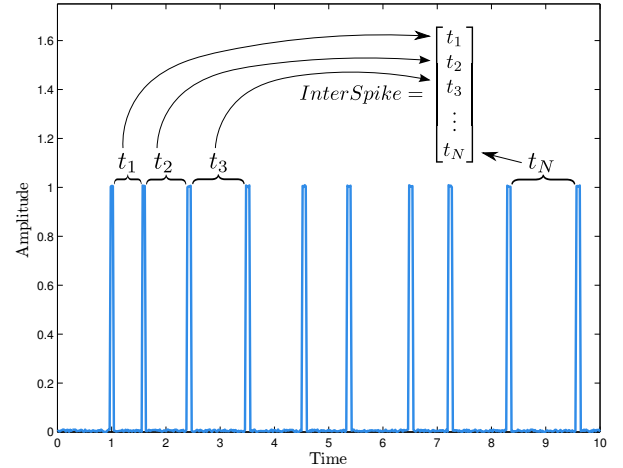


Figure 5. Time signal of an excitable circuit. The inactivity intervals are measured and saved to then quantify the phenomena.

corresponding to the signal's period, this maximum is plotted with respect to the noise and it is obtained the typical characteristic resonance curve.

Another useful way to quantify SR is through the Fourier Transform of the input signal, this transformation will exhibit peaks at the multiple frequency values of the input signal. Once the peaks are found, the peak that corresponds to the input frequency is integrated and this peak is plotted with respect to the noise amplitude.

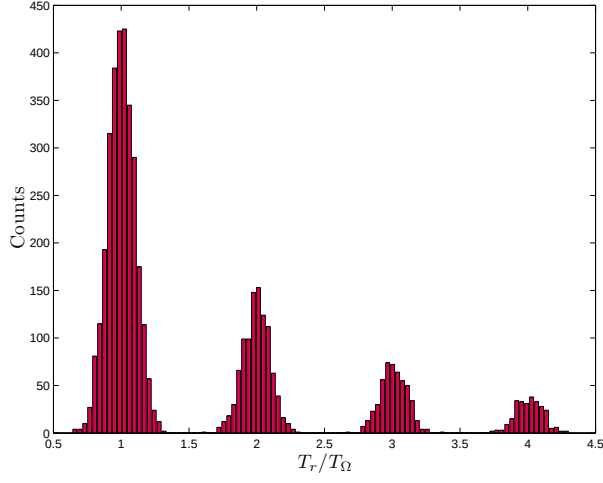


Figure 6. The inactivity time data that was collected are now organised in a histogram which relates the inactivity time with the period signal.

The integral for this calculation is presented in eq.4⁷.

$$SNR = \left[\lim_{\Delta\omega \rightarrow 0} \int_{\Omega - \Delta\omega}^{\Omega + \Delta\omega} S(\omega) d\omega \right] \quad (4)$$

III. METHODOLOGY

In the following section it is presented the general descriptions of the neural model in which SR is analysed. The circuit used is showed in fig. 7. It is mainly based in a Schmit Trigger monostable, this is a device with a non dynamical threshold that selects the signals that are given to it as input through comparison with an intrinsic threshold. If the input signals overcome the intrinsic threshold, a spike is emitted. This Schmit Trigger simulate pretty well the behaviour of a neuron in its task of transmit signals through pulsed or action potentials fig.8

In fig.7, there is not only one Schmit Trigger but two defined as U1 and U2. U1 is an operational amplifier that receives the input signal and amplifies it in order for this signal to feed the first monostable (1B). It is this first monostable which shoot or not shoot depending on the received signal, this means that it selects between the given input data and when it finds one that overcomes its intrinsic threshold, an spike is produced which causes that U2 generates a pulse of duration (T1) that can vary according to the desired resistance and capacitance values.

Is the behavior of U2 the one which simulates a neuron in its active spike emission stage. When the pulse

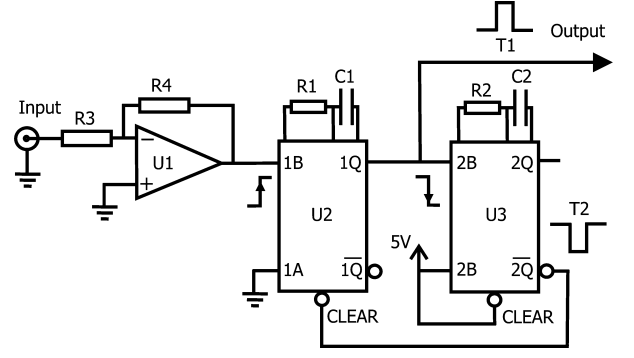


Figure 7. Artificial neuron circuit. Based on monostable Schmit Trigger⁸

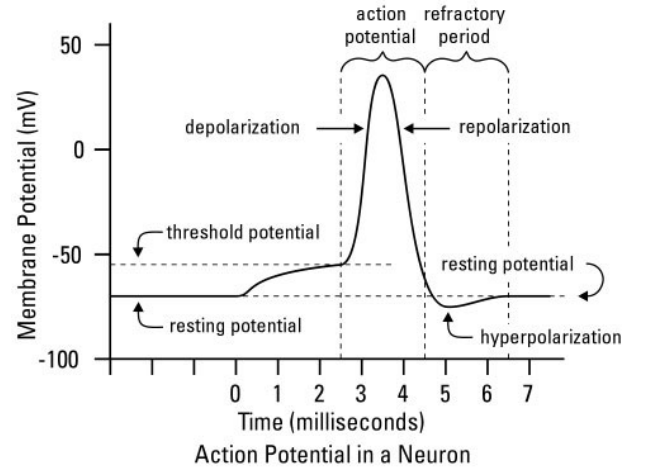


Figure 8. Action potential of a neuron. The neuron has a threshold and emits spikes when its threshold value is overcome. Spikes would be limited by the refractory time of the neuron⁹.

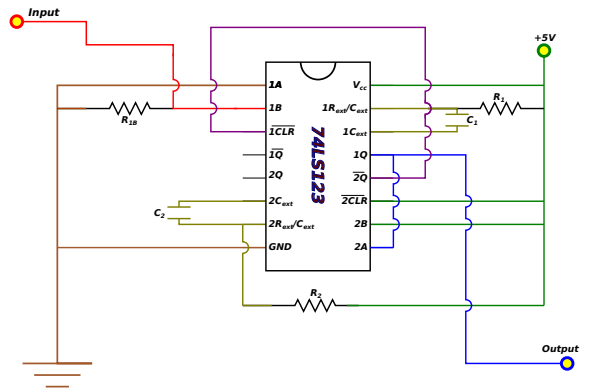


Figure 9. Artificial neuron as should be assembled to work properly.

of U2 enters in descendent flank, the second monostable is fired (2B) with a duration (T2) which is the same adjustable mode by capacitances and resistance. The output $2\bar{Q}$ of U3 works as clear of the first monostable, it inhibits in this way future spikes until the descendent flank does not exist anymore. This section of the circuit is the one which simulates the refractory period of a neuron. Once a cycle is completed, it is repeated indefinitely until it is not stimulated.

The complete experimental setup to measure SR is shown in fig. 10, the coherent signals generator emits a periodic output that is added to the noise generated by the noise source. The addition of these signals is the one which enters the neural circuit. This circuit is going to generate an output if the signals overcome its intrinsic threshold. This output is connected to an oscilloscope that save the data that is going to be processed later.

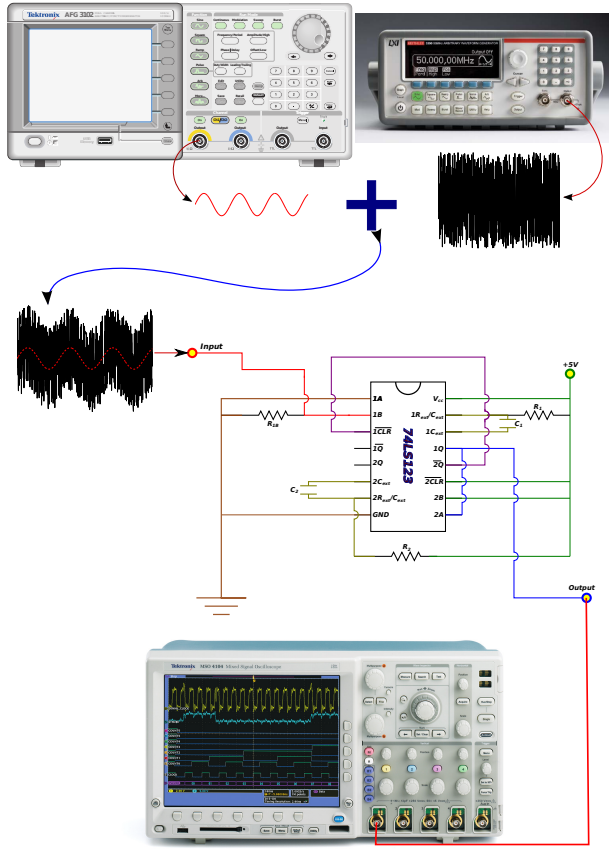


Figura 10. Experimental setting for Studying Stochastic Resonance, on the upper left is the coherent signal generator, in the upper right the White Noise generator, at the center is the artificial Neuron, and at the bottom is the oscilloscope which was used for data acquisition.

The color noise variation is developed by the creation of a pink noise generator. It is not easy to obtain pink noise but after coupling two filters as is shown in fig.11, it is achieved the generation of pink noise. The filters

reduce the high frequency content of white noise, this means that the white noise source is required and another filter is added to produce pink noise. For the confirmation of the correct acquisition of pink noise, it is necessary to change the basis of the time signal to the power spectrum and find there the characteristic slope in the logarithmic scale.

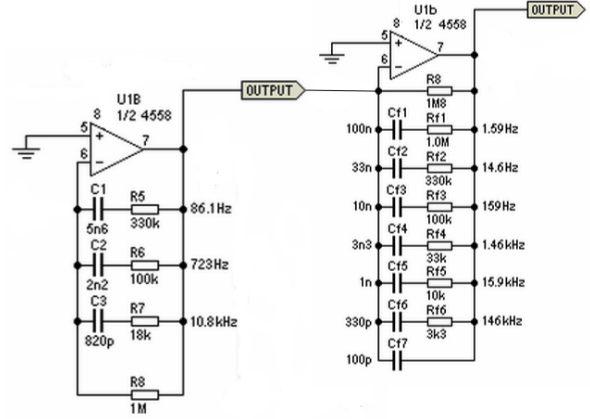


Figura 11. Filtro de altas frecuencias específico para la obtención de ruido pink

IV. RESULTS AND DISCUSSION

In the following section are described the obtained results in the SR characterisation, in parallel are the comparisons between the obtained results for pink and white noise.

A. Power spectrum

A Fourier Transform should be done to the output signals of the circuit in order to obtain the power spectrum. It is necessary to clarify that the Fourier transforms were obtained from a program done in MATLAB but they are not going to be shown here given that the obtained numerical results are not as accurate as what was obtained through the spectrum analyser. In the following section are presented the obtained spectrum.

The x axis corresponds to the frequency and the y axis corresponds to the power. It is the x axis the one that indicates if the maximum peak of the spectrum matches with the input signal frequency, which happens a long all the spectrums from the small, optimum and high noise values.

In the fig.13 and 12 are the corresponding power spectrum for white and pink noise, it is observed a difference in the number of spike's events, where it is more often for the case of Pink noise.

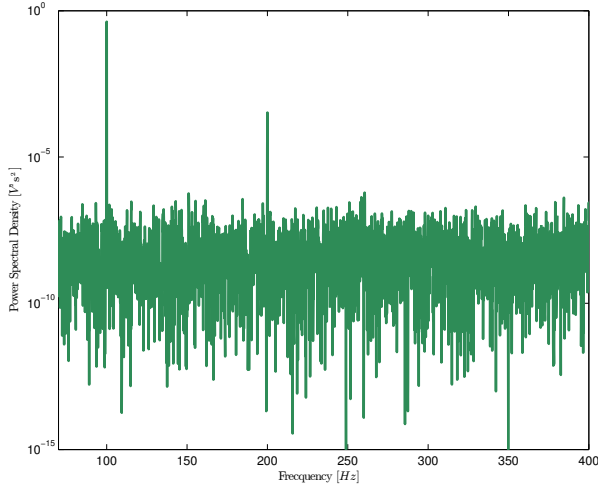


Figure 12. Fourier transform of the output signal working with a white noise amplitude of 2500mV.

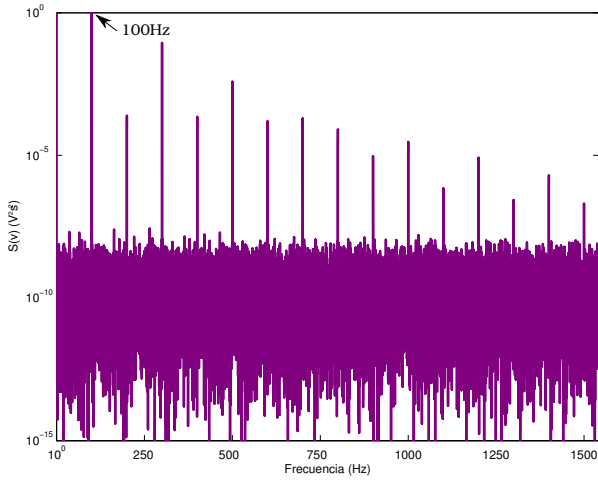


Figure 13. Fourier transform of the Output signal working with a Pink noise amplitude of 2900mv.

It is necessary to clarify in the fact that as long as the noise value is increased, the spectrum amplitude also increase but reaches a point in which no matter how much is increased the voltage of the pink noise, the spectrum reaches a maximum value and start descending.

Even if it is presented many plots, only one shows the number of the total spectrum that should be achieved to achieve a SNR curve, given that is from the spectrums that it is achieved the final characterisation, this is explained in the following section.

B. SNR

Recapitulating the observed a long the given spectrums by fig.12 and the fig.13, it is clear that every time, a maximum peak is conserved along the spectrums. This peak is selected and it is integrated taking in a vector the values that are found by the integral together with the respective noise voltage value correspondent to each peak. Afterwards the vectors are plotted and the SNRs are obtained as shown in fig.14 and fig.15.

These SNR correspond to the spectrums shown in the last section where it was shown that the 100Hz of the input signal are kept a long all spectrums. The plots shown here are amplifying the input signal for values of white and pink noise respectively.

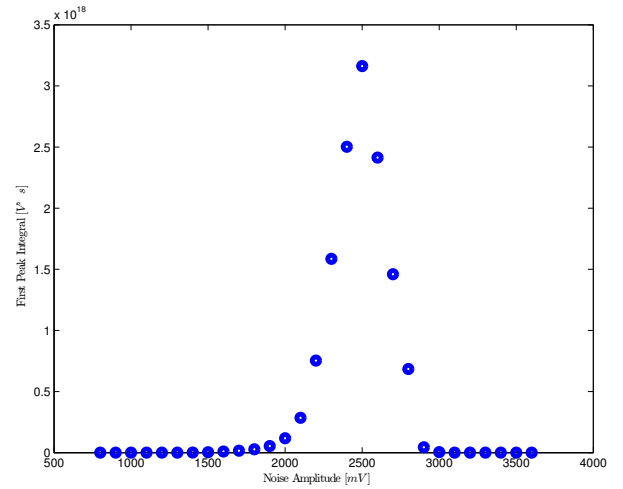


Figure 14. Signal to Noise Ratio for white noise, coherent signal of 50mV.

It is observed an important difference between the two noise colors in the fact that even if the measurements were made exactly for the same voltages and under the same parameters, the SR maximum is achieved for different optimum noise values. Besides, the range of noise values that make the SR maxima is bigger for the case of pink noise than for the case of white noise where the range values is sharper.

C. Noise color comparison

When the power spectrum obtained for white and pink noise is compared, it is observed a big difference in the relevance that each noise input makes to the input signal frequency that is found in the 100Hz, this difference is of the order of 100 times bigger for the contribution made by white noise as is presented in fig.?? a and b.

This is a surprising finding because of the fact that as

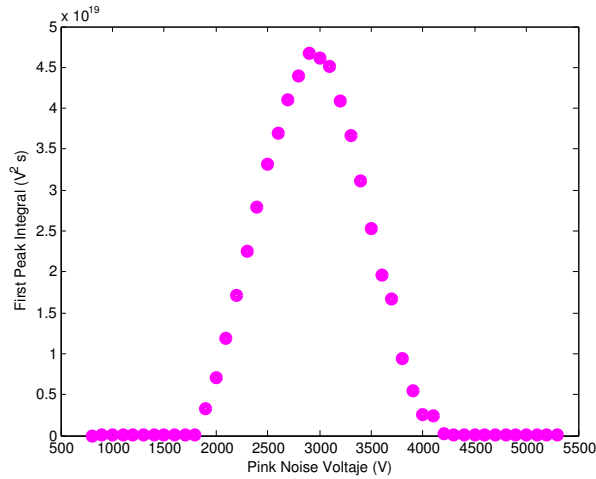


Figura 15. Signal to Noise Ratio for pink noise, coherent signal of 50mv.

seen in fig.?? c, even if the contribution of Pink noise close to the frequency range of the input signal is 100 times less than in the white noise case, this is amplifying the weak input signal close to 20 times more than in the case of white noise.

This result is unexpected but with huge consequences given that a big number of physical phenomena and biological are immerse in a pink noise environment, and we proof its amplification power which explains its relevance for nature and at the intrinsic mechanisms of measurement devices. It is important to notice that theoretical models, numerical and computational that are currently implemented, made use of white noise given the fact that there is a mathematical algorithm to generate it, while for the case of pink noise, there is no a mathematical algorithm that is able to generate it. After the finding presented here, it is evident the importance of pink noise and should be implemented in physical models given that is more realistic and twenty time more optimum than white noise in it amplification characteristics.

It is important to mention that there is no bibliography that mentions this relevant finding and it is declare that we are the first who found it and report it.

V. CONCLUSIONS

It was achieved the detection of the Stochastic Resonance phenomena for an artificial neuron, even if this detection is not easy and required of many trials (explained by the uncertainty in the use of noise), it was possible to make many replicas of the same result, which assure the accuracy of the finding.

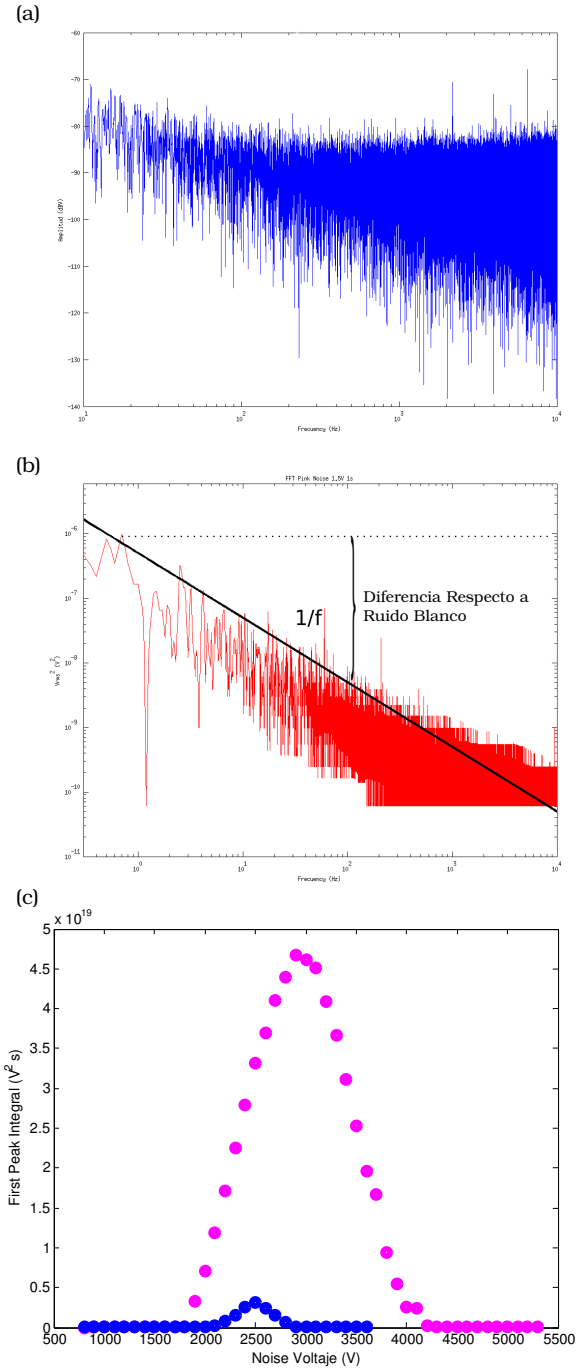


Figura 16. (a) Fourier Transform of white noise, (b) Fourier Transform of Pink noise

As method for SR characterisation is better to use the SNR which requires the integration of the maximum peaks of the power spectrums given that the Inter Spike Interval Histograms are harder to program and the results are not as precise as in the case of the SNR. It is also recommendable no to program the Fourier Transform of the signal in time given that there are complications to program the transform and under slight parameter variations, the results change rapidly, while

for the case in which the transform is obtained directly from the spectrum analyzer directly, it is faster and more reliable.

It was proved that the SR phenomena amplifies by the use of noise weak periodic signals. The SR conserves the information of such weak periodic signal contrary to what intuition could say by the fact that the signal is embedded in a noisy environment with the content of all frequency values and with higher amplitudes than the desired signal. Noise helps to detect this signal. This detection of the frequency of the input signal and its even multiples was present in all measurements and was also replicated, validating the reliability of the results.

As long as time passed, the voltage of the input signal was decreased from 800mV, 500mV, 300mV,... until the 40mV value, which confirm the power of the SR to enhance and conserve small signals.

Finally, the most important conclusion is that the

artificial neuron have the SR phenomena in white and pink noise but the variation to pink noise shows the enhancement in the amplification. Pink noise has a bigger bandwidth allowing more optimum values to obtain the maximum, while white noise has a most punctual optimum value. With this result it is understandable that neurons are more sensible to the detection of signals that carry pink noise instead to those without noise of with white noise. This is not only conclusive as neural model but also can be a model for all kinds of setups with threshold behavior, excitability and where detection of small signals embedded in high amplitude noise is desirable.

A last remark with respect to the possible implications for our perception of signals by sensory neurons is that it is limited by the external and intrinsic neuronal noise. In this way, if we add pink noise to the high number of signals that are not easily detected by our senses, then SR is possible in a natural way and extra-sensorial capabilities are created in a natural way.

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